



# Principles of Equation-Based Object-Oriented Modeling and Languages

Module D: Co-simulation and the Functional Mock-up Interface

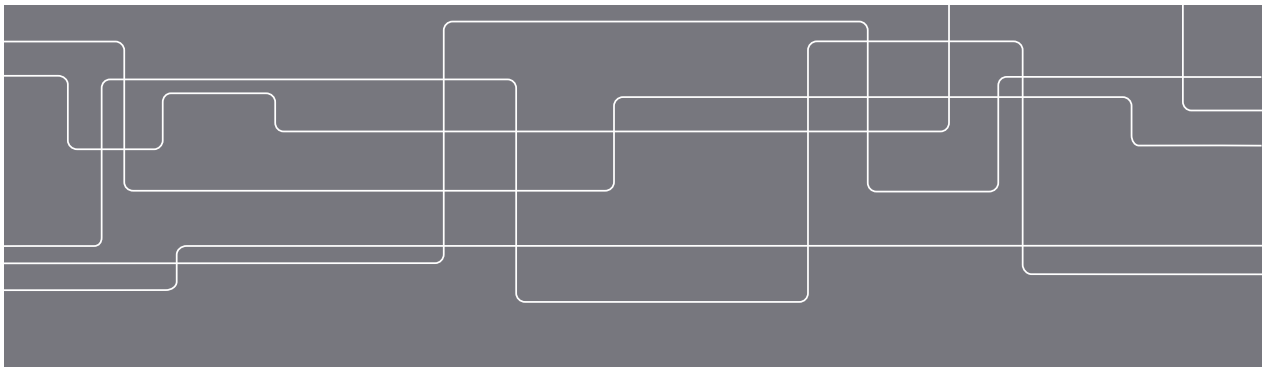
Mini-course, Scuola Superiore Sant'Anna, Pisa, Italy.

December 9-10, 2014

**David Broman**

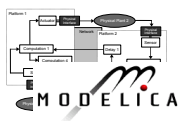
Associate Professor, KTH Royal Institute of Technology

Assistant Research Engineer, University of California, Berkeley



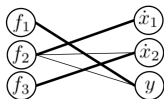
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## Course Structure



### Module A

EOO Languages and Modelica Fundamentals



### Module B

DAEs and Algorithms in EOO Languages



### Module C

Modelyze – Defining Equation-Based DSLs



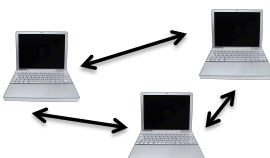
### Module D

Co-simulation and the Functional Mock-up Interface

# Agenda

**Part I**

**Introduction to FMI**



**Part II**

**FMI Formalization and Master Algorithms**

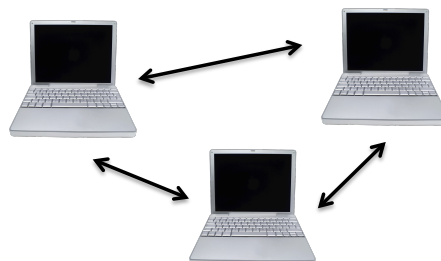
$$\begin{aligned} \text{init}_c &: \mathbb{R}_{\geq 0} \rightarrow S_c \\ \text{set}_c &: S_c \times U_c \times \mathbb{V} \rightarrow S_c \\ \text{get}_c &: S_c \times Y_c \rightarrow \mathbb{V} \\ \text{doStep}_c &: S_c \times \mathbb{R}_{\geq 0} \rightarrow S_c \times \mathbb{R}_{\geq 0} \end{aligned}$$

David Broman  
dbro@kth.se

**Part I**  
Introduction  
to FMI

**Part II**  
FMI Formalization  
and Master Algorithms

## Part I Introduction to FMI

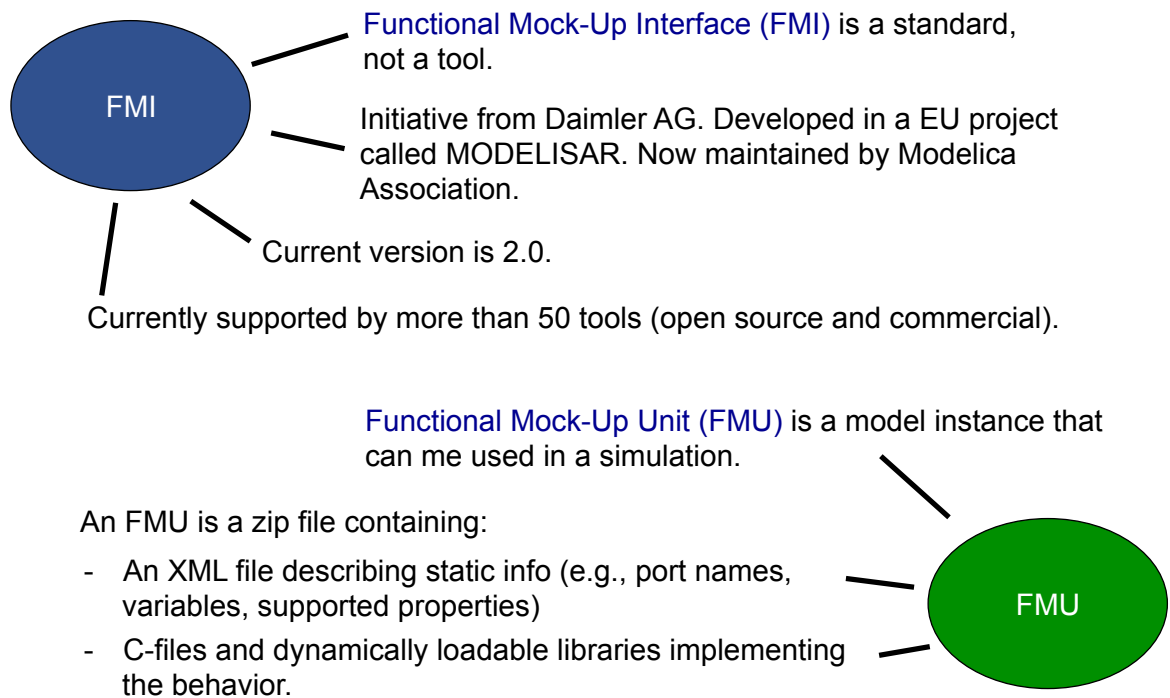
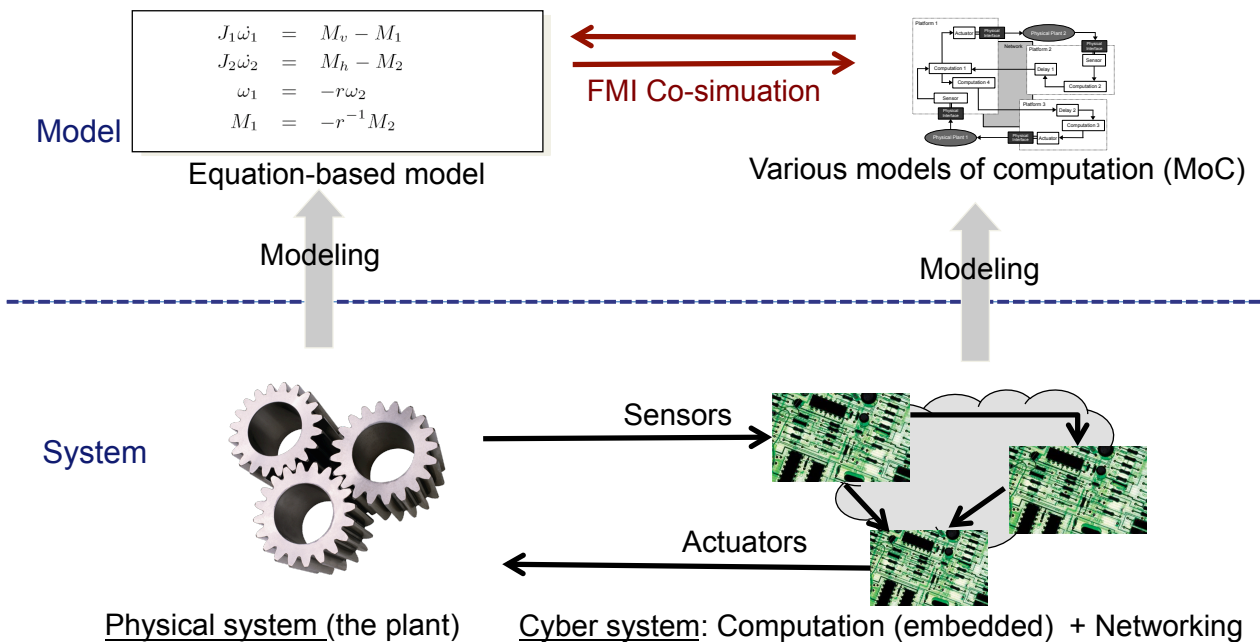


David Broman  
dbro@kth.se



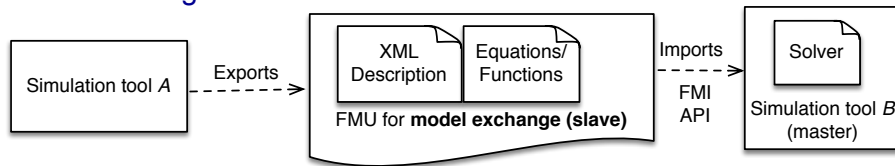
**Part I**  
Introduction  
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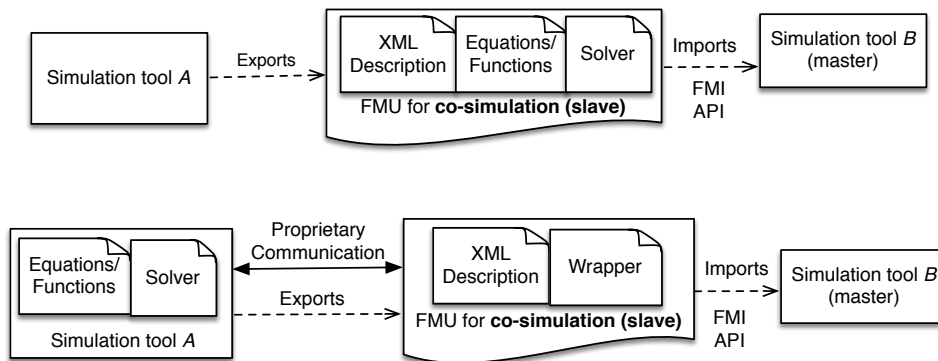


# Model Exchange or Co-Simulation?

## FMI for Model Exchange



## FMI for Co-Simulation



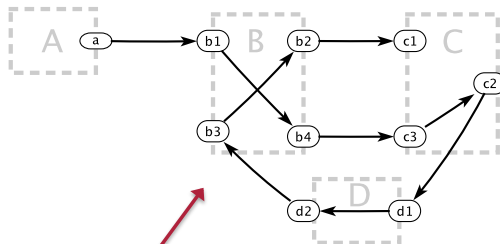
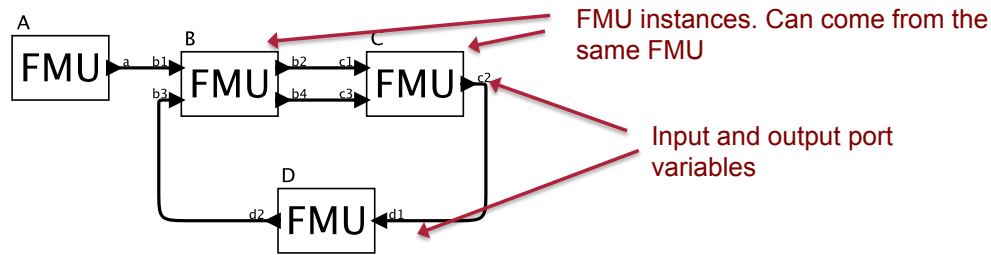
## Part II FMI Formalization and Master Algorithms

$$\begin{aligned} \text{init}_c &: \mathbb{R}_{\geq 0} \rightarrow S_c \\ \text{set}_c &: S_c \times U_c \times \mathbb{V} \rightarrow S_c \\ \text{get}_c &: S_c \times Y_c \rightarrow \mathbb{V} \\ \text{doStep}_c &: S_c \times \mathbb{R}_{\geq 0} \rightarrow S_c \times \mathbb{R}_{\geq 0} \end{aligned}$$



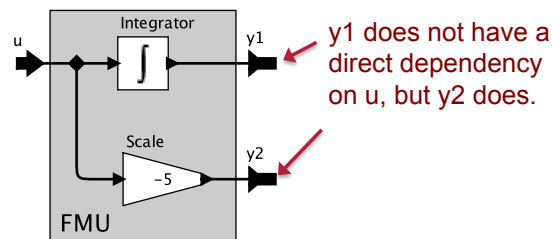


# FMU Connections and I/O Dependencies

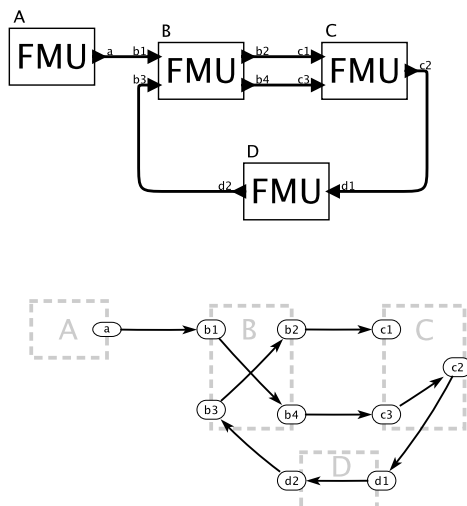


I/O dependencies together with port connections form a graph.

Can output be directly dependent on the input?



## A FMI formalization (a subset of the standard)



Set of FMU instances in a model  
FMU instance identifier

$C$   
 $c \in C$

Set of state valuations for instance  $c$

$S_c$

Set of input port variables for instance  $c$

$U_c$

Set of output port variables for instance  $c$

$Y_c$

Set of values that a variable may take on

$\mathbb{V}$

I/O dependency for instance  $c$

$D_c \subseteq U_c \times Y_c$

Set of all input variables in a model

$U = \bigcup_{c \in C} U_c$

Set of all output variables in a model

$Y = \bigcup_{c \in C} Y_c$

Set of all I/O dependencies

$D = \bigcup_{c \in C} D_c$

Port mapping

$P : U \rightarrow Y$

## A FMI formalization (subset of standard)

$\text{init}_c : \mathbb{R}_{\geq 0} \rightarrow S_c$	Set of FMU instances in a model	$C$
$\text{set}_c : S_c \times U_c \times \mathbb{V} \rightarrow S_c$	FMU instance identifier	$c \in C$
$\text{get}_c : S_c \times Y_c \rightarrow \mathbb{V}$	Set of state valuations for instance $c$	$S_c$
$\text{doStep}_c : S_c \times \mathbb{R}_{\geq 0} \rightarrow S_c \times \mathbb{R}_{\geq 0}$	Set of input port variables for instance $c$	$U_c$
	Set of output port variables for instance $c$	$Y_c$
	Set of values that a variable may take on	$\mathbb{V}$
	I/O dependency for instance $c$	$D_c \subseteq U_c \times Y_c$
	Set of all input variables in a model	$U = \bigcup_{c \in C} U_c$
	Set of all output variables in a model	$Y = \bigcup_{c \in C} Y_c$
	Set of all I/O dependencies	$D = \bigcup_{c \in C} D_c$
	Port mapping	$P : U \rightarrow Y$

(A0) If  $\text{doStep}_c(s, h) = (s', h')$  then  $0 \leq h' \leq h$ .

(A1) If  $\text{doStep}_c(s, h) = (s', h')$ , then for any  $h''$  where  $0 \leq h'' \leq h'$ ,  $\text{doStep}_c(s, h'') = (s'', h'')$  for some  $s''$ .

← If  $h' < h$ ,  $\text{doStep}$  rejected  $h$ . In such a case “roll-back” is needed.

## FMI 2.0 restrictions not seen in previous versions

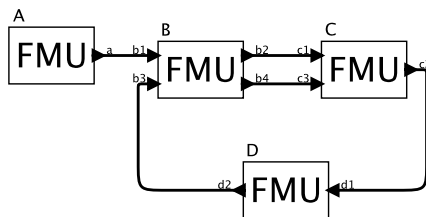
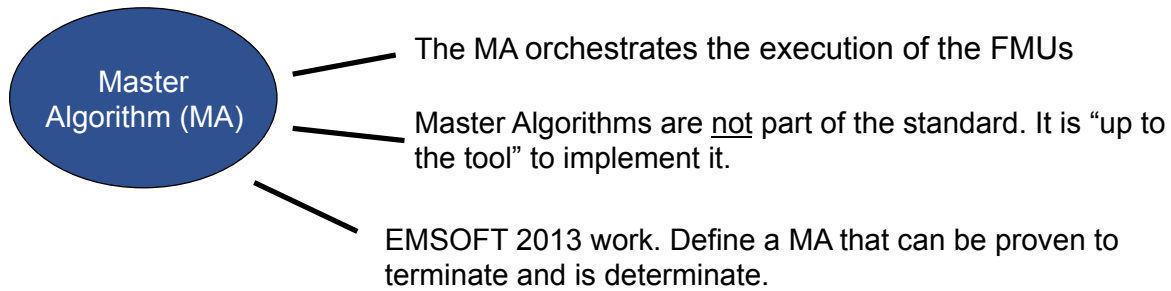
$\text{init}_c : \mathbb{R}_{\geq 0} \rightarrow S_c$   
 $\text{set}_c : S_c \times U_c \times \mathbb{V} \rightarrow S_c$   
 $\text{get}_c : S_c \times Y_c \rightarrow \mathbb{V}$   
 $\text{doStep}_c : S_c \times \mathbb{R}_{\geq 0} \rightarrow S_c \times \mathbb{R}_{\geq 0}$

Version 2.0 makes it impossible to implement a component with zero latency (this was not a restriction in previous versions!)

“There is the additional restriction in “slaveInitialized” state that it is not allowed to call `fmi2GetXXX` functions after `fmi2SetXXX` functions without an `fmi2DoStep` call in between.”  
**(FMI standard 2.0, July 25, 2014, page 104)**

“... communication step size ( $h_c$ ). The latter must be  $> 0.0$ ”  
**(FMI standard 2.0, July 25, 2014, page 100)**

For FMI to support **hybrid co-simulation**, we believe that this restriction was a major mistake. We hope that future versions will improve this situation...




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**Algorithm 1:** *Order-Variables.*


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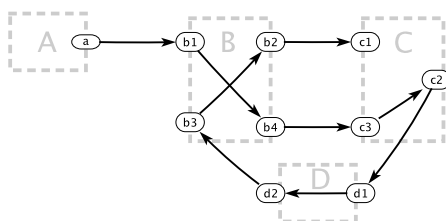
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**Input:** Port mapping  $P$ , global dependency relation  $D$ , and global set of variables  $\mathbb{X}$ .  $\mathbb{X} = U \cup Y$

**Output:** An ordered list  $\bar{x}$  of variables, or error.

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1. Let  $G$  be a directed graph, where the vertices are represented by port variables  $\mathbb{X}$  and an edge  $e \in \mathbb{X} \times \mathbb{X}$  is a variable dependency. The set of all edges  $E$  is then constructed by  $E = D \cup \{(y, u) \mid u \in U \wedge P(u) = y\}$ .
2. Perform a topological sort on  $G$ . If a cycle in  $G$  is found, terminate and return error. If no cycles are found, the resulting list of variables is  $\bar{x}$ .



## Algorithm 2: Master-Step

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**Algorithm 2:** *Master-Step.*


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**Input:** Set of instances  $C$ , ordered variable list  $\bar{x}$ , port mapping  $P$ , the maximal step size  $h_{max}$ , and a mutable state mapping  $m$  of size  $|C|$ .

**Output:** Updated state mapping  $m$  and the performed step size  $h$ .

---

1. Set values for all input variables:  
For each  $u \in \bar{x}$  (in order) where  $u \in U$  do
  - (a)  $y := P(u)$
  - (b)  $v := \text{get}_{c_y}(m[c_y], y)$
  - (c)  $m[c_u] := \text{set}_{c_u}(m[c_u], u, v)$
2. Save the states of all FMUs to enable rollback:  
 $r := m$
3. Set communication step size to an initial default value:  
 $h := h_{max}$
4. Find  $h$  acceptable by all FMUs:  
For each  $c \in C$  do
  - (a)  $(s', h') := \text{doStep}_c(m[c], h_{max})$
  - (b)  $h := \min(h, h')$
  - (c)  $m[c] := s'$
5. Assert  $0 \leq h \leq h_{max}$  // follows from Assumption (A0)
6. If  $h < h_{max}$  then // roll back and perform step  $h$   
For each  $c \in C$  do
  - (a)  $(s', h') := \text{doStep}(r[c], h)$
  - (b) Assert  $h' = h$  // follows from Assumption (A1)
  - (c)  $m[c] := s'$
7. Return  $m$  and  $h$ .

## Predictable Step Size

Problems with Algorithm 2

- Requires that all FMUs support rollback
- Inefficient if the FMU can predict the next event.

Propose to extend FMI with one new function:

$\text{getMaxStepSize}_c : S_c \rightarrow \mathbb{R}_{\geq 0} \cup \{\infty\}$

Returns the upper bound of the communication step-size that an FMU can accept

$C_P$ : Set of Predictable FMU Instances

$C_R$ : Set of FMUs with rollback capabilities

$C_L$ : Set of Legacy FMUs that neither implement rollback, nor are predictable.

(A4) If  $c \in C_P$  and  $s \in S_c$  and  $\text{getMaxStepSize}_c(s) = h$  then for all  $h'$  where  $0 \leq h' \leq h$ ,  $\text{doStep}_c(s, h') = (s', h')$  for some  $s'$ .

- (a)  $|C_L| \leq 1$ .
- (b)  $C_L \cup C_R \cup C_P = C$ .
- (c)  $C_L \cap C_R = \emptyset$  and  $C_R \cap C_P = \emptyset$  and  $C_P \cap C_L = \emptyset$ .

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**Algorithm 3: Master-Step With Predictable Step Sizes.**


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**Input:** Set of instances  $C$ , ordered variable list  $\bar{x}$ , port mapping  $P$ , the maximal step size  $h_{max}$ , and a mutable state mapping  $m$  of size  $|C|$ .

**Output:** Updated state mapping  $m$  and the performed step size  $h$ .

---

1. Set values for all input variables:  
For each  $u \in \bar{x}$  (in order) where  $u \in U$  do
  - (a)  $y := P(u)$
  - (b)  $v := \text{get}_{c_y}(m[c_y], y)$
  - (c)  $m[c_u] := \text{set}_{c_u}(m[c_u], u, v)$
2. Find the minimal predictable communication size:  
 $h := \min(\{\text{getMaxStepSize}_c(m[c]) \mid c \in C_P\} \cup \{h_{max}\})$
3. Save the states for all instances that can perform rollback:
  - (a) For each  $c \in C_R$  do  $r[c] := m[c]$
  - (b)  $\text{doStepOnLegacy} := \text{true}$
  - (c) Goto step 5.
4. Restore states for rollback instances.  
For each  $c \in C_R$  do  $m[c] := r[c]$
5. Perform **doStep** on all instances with rollback:  
 $h_{min} := h$   
For each  $c \in C_R$  do
  - (a)  $(s', h') := \text{doStep}_c(m[c], h)$
  - (b)  $h_{min} := \min(h', h_{min})$
  - (c)  $m[c] := s'$
6. If  $h_{min} < h$  then  $h := h_{min}$  and goto step 4.
7. Perform **doStep** on the legacy FMU (if it exists)  
If  $c \in C_L$  and  $\text{doStepOnLegacy}$  then
  - (a)  $(s', h') := \text{doStep}_c(m[c], h)$
  - (b)  $m[c] := s'$
  - (c)  $\text{doStepOnLegacy} := \text{false}$
  - (d) If  $h' < h$  then  $h := h'$  and goto step 4.
8. Perform **doStep** on all FMUs with predictable step size:  
For each  $c \in C_P$  do
  - (a)  $(s', h') := \text{doStep}_c(m[c], h)$
  - (b) Assert  $h' = h$  // follows from Assumption (A4)
  - (c)  $m[c] := s'$
9. Return  $m$  and  $h$ .

- Blockwitz et al. Functional Mockup Interface 2.0: The Standard for Tool independent Exchange of Simulation Models, In Proceedings of the 9th International Modelica Conference, 2012  
(General overview of an early version of 2.0, not the final release)
- David Broman, Christopher Brooks, Lev Greenberg, Edward A. Lee, Michael Masin, Stavros Tripakis, and Michael Wetter. Determinate Composition of FMUs for Co-Simulation. In *Proceedings of the International Conference on Embedded Software (EMSOFT 2013)*, Montreal, Canada, 2013.  
(Defines the master algorithm and formalization that we presented here)
- David Broman, Lev Greenberg, Edward A. Lee, Michael Masin, Stavros Tripakis, and Michael Wetter. *Requirements for Hybrid Cosimulation*, EECS Technical report No. UCB/EECS-2014-157, UC Berkeley, August 14, 2014.  
(A document describing requirements and test cases for hybrid co-simulation)

## Summary and Conclusions



## Summary and Conclusions

### Some key take away points:

- The **FMI standard** is used in both industry and academia.
- There are two main modes: FMI for **model exchange** and FMI for **co-simulation**.
- We have presented a **formalization** of a core of FMI and proposed master algorithms that are determinate.
- Note, however, that the latest FMI standard for co-simulation, have made **it impossible to encode such hybrid co-simulation correctly**. We hope that we can change this in the next version of the FMI standard.



Thanks for listening!